Comment on "First Order Transition in the Ginzburg-Landau Model"

In a recent Letter, Curty and Beck [1] have shown very interesting results which indicate that the Ginzburg-Landau (G-L) transition becomes first order when the coherence length $\xi = \xi_0 |t|^{-\frac{1}{2}}$ ($t \equiv T/T_0$-1 is the reduced temperature) becomes of the order of the lattice spacing ε. They considered the lattice G-L model parametrized by: $\sigma \equiv \frac{\varepsilon^2}{\xi_0^2}$, which controlls the strength of amplitude fluctuations (that grow as σ decreases), and $V_0 \equiv \frac{1}{k_B}\frac{a}{b}\gamma$, which governs the overall strength of the complex G-L field $\psi = |\psi| \exp[i\theta]$. They treated the model by a variational approximation and got a criterion for first order transition in the form of an inequality which involves the spatial dimension d. They concluded that their criterion for $d = 2$ is not clear and that doubt remained if the first order found for this case is not an artifact of the used approximation.

Here I will present clear evidences that for d=2 a first order transition takes place when ξ becomes $\sim \varepsilon$ ($\sigma \sim 1$) and that this is connected with a sudden proliferation of vortices. Similar results where reported in [2] although using a different parametrization of the G-L model which obscures the comparison with [1]. The $d = 2$ G-L hamiltonian $H[\sigma, V_0]$ was simulated on $L \times L$ lattices. The measured phase diagram on the plane $\tilde{T}$-$\tilde{\sigma}^{-\frac{1}{2}}$ is showed in Fig.1 where, as in ref. [1], $\tilde{T} = T/(-tV_0)$ and $\tilde{\sigma} = -t\sigma$.

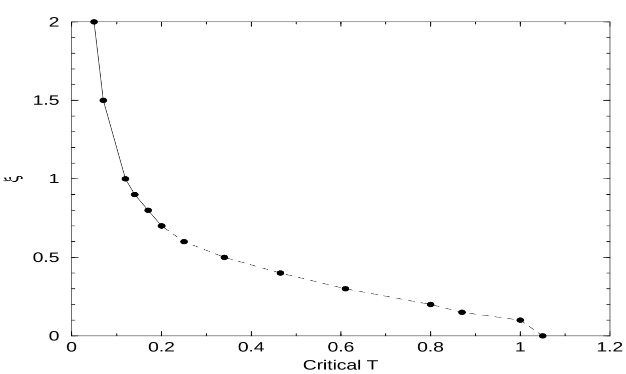

Figure 1: Phase diagram in the $(\tilde{T}, \tilde{\sigma}^{-\frac{1}{2}})$ plane for $L = 40$.

Above $\xi/a \equiv \tilde{\sigma}^{-\frac{1}{2}} = 0.8$ the phase transition line changes from second order (dashed line) to first order (filled line). The double peak of the energy density e histogram corresponding to the two coexisting phases, characteristic of a first-order transition, is showed in Fig. 2-a for $\tilde{\sigma}^{-\frac{1}{2}} = 0.85$. Both peaks remain fixed as L increases and the width of each of them clearly scales as $\sqrt{(\frac{1}{L^D})} = \frac{1}{L}$, due to ordinary non-critical fluctuations (Fig. 2-b). In addition, a strong hysteresis effect was found for e when considering heating and cooling runs. On the other hand, for $\tilde{\sigma}^{-\frac{1}{2}} \leq 0.75$ the peaks are much lower and wider, they move towards to an intermediate

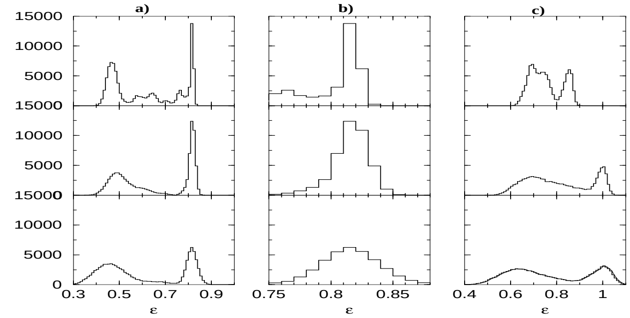

Figure 2: Histograms of e for L=10 (below), L=20 (middle) and L=40 (above). a) $\tilde{\sigma}^{-\frac{1}{2}} = 0.85$. b) Zoom of the right peak. c) $\tilde{\sigma}^{-\frac{1}{2}} = 0.75$.

value of e as L increases and their width do not scale as $\frac{1}{L}$ (Fig. 2-c). Moreover, no hysteresis in e is found.

The central role played by vortex excitations in determining the nature of the phase transition can be seen in Fig. 3-a where v is plotted vs. $\tilde{T}$ for different values of $\tilde{\sigma}$ and L=40. For $\tilde{\sigma}$=1 there is a clear discontinuity in the vortex density v (Fig. 3-b). As long as $\tilde{\sigma}$ increases the jump becomes more smooth and moves to higher values of $\tilde{T}_c$ until for $\tilde{\sigma} = 100$ one gets something very close to the Kosterlitz Thouless (K-T) behavior of the XY model. The enhancement of vortex production when amplitude fluctuations are large is due basically to the fact that they decrease the energy of vortices.

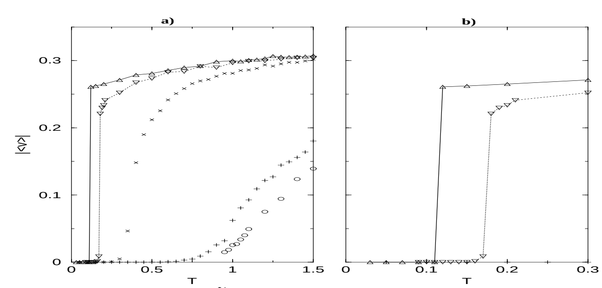

Figure 3: a) v vs. $\tilde{T}$ for $\tilde{\sigma}$=1 ($\triangle$), $\tilde{\sigma}$=1.5625 (∇), $\tilde{\sigma}$=4 ($\times$), $\tilde{\sigma}$=100 (+), XY model(o). b) Zoom of 3-(a)

Therefore, in the G-L model the nature of the phase transition depends dramatically on the value of $\tilde{\sigma}$: For $\tilde{\sigma}$ <1.5625 ($\frac{\xi}{\varepsilon} > 0.8$) the density of vortices experiments a discontinuous jump which coincides with a first order transition. On the other hand, for $\tilde{\sigma} \gg 1$ the G-L reduces to the XY model with the more subtle K-T phase transition.

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PACS numbers: 74.25.Dw, 64.60.-i, 05.70.Fh

References

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[2] G. Alvarez and H. Fort, cond-mat/0006341, submitted to Phys. Rev. Lett..